On the interpretation of Farrell efficiency measures and the Malmquist productivity index

by

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Abstract: There is a close connection between efficiency and productivity: efficiency can be defined as the relation between observed productivity and a benchmark for a potentially better productivity level that may be obtained. Within the research strand of non-parametric frontier functions the 'ratio' definition of efficiency in Charnes et al. (1978) of weighted outputs on weighted inputs has been seen in this light. However, a productivity interpretation of output weighted with endogenous shadow prices divided by inputs weighted with another set of endogenous shadow prices should not be interpreted as a productivity measure. Reasons are that the measuring unit of the output weights are efficiency score units per output, the weighted sum of the outputs is restricted to be between zero and one, and the sum of the weighted inputs is normalized to unity. An additional anomaly is that some weights may be zero.

The radial Farrell (1957) measures of efficiency expressed on 'envelopment' form can be shown to have the basic property of an efficiency definition of comparing observed productivity with a benchmark for productivity.

The Malmquist productivity index differs from the standard productivity indices of the national accounts using market prices to weigh together outputs and inputs. Prices are not used when calculating the index. Measuring productivity change is based on treating all outputs equal from a value point of view, and the same holds for inputs. Instead of some sort of value weights the extent to which inputs for given outputs (input orientation) can be maximally reduced proportionally for one period if the most efficient technology is employed, relative to the maximal proportional reduction in a previous period using the same technology. An output-oriented productivity-change index for given inputs can be set up in an analogous way. In order to function as a TFP index the production possibility set used must be linear homogenous and the Farrell efficiency measures used must have the proper homogeneity properties. Extending the Farrell constant returns to scale efficiency measures to cover variable returns to scale as done in Førsund and Hjalmarsson (1974); (1979) the measure of technical productivity is of especial interest when understanding the Malmquist productivity index.

A popular use of the Malmquist productivity index is to decompose it into efficiency change and frontier-shift change. However, it is argued in the paper that this decomposition in many cases is misused putting casual explanatory power on the decomposition parts. The efficiency change is not a measure of what efficiency change has contributed to productivity change, but a measure of the relative change in the distance from the frontier. Likewise, the frontier shift as measured in the decomposition shows the relative potential for productivity increase due to

technical change, and not the actual contribution. It is argued that a casual decomposition cannot be identified based on the information used to estimate the Malmquist productivity index.

References

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